Compactifications of moduli spaces and stratified homotopy theory

Mikala Jansen

Compactifications of locally symmetric spaces or more generally moduli spaces often come equipped with natural stratifications, that is, a "well-behaved" partition of the space. Concrete examples include the Borel–Serre and reductive Borel–Serre compactifications of the locally symmetric space associated to an arithmetic group, and the Deligne–Mumford–Knudsen compactification of the moduli stack of stable curves. Arising from this additional structure are a wealth of interesting constructible (complexes of) sheaves, i.e. sheaves which are locally constant along each stratum (but not necessarily on the whole space!). These in turn define interesting cohomology theories, e.g. intersection cohomology and weighted cohomology.

It is a classical result that locally constant sheaves on a sufficiently nice topological space are classified by the fundamental groupoid, or homotopy type. For stratified spaces, we have a similar classification of constructible sheaves as representations of the so-called exit path category, or stratified homotopy type. Calculating the stratified homotopy type of a concrete stratified space would allow us to study the constructible sheaves from a more combinatorial viewpoint – in theory at least.